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hep-lat/9708001**SIMULATIONS OF THE ELECTROWEAK PLASMA AT  
FINITE TEMPERATURE <sup>a</sup>**

Joachim Hein

*Dept. Physics & Astronomy, University of Glasgow, Glasgow G12 8QQ, UK*

We investigate the SU(2) Higgs model at a Higgs Boson mass of  $\simeq 34$  GeV for temperatures at the electroweak scale. We discuss in detail the critical temperature, the scalar field vacuum expectation value and the latent heat. We also consider for which temperatures the plasma can be regarded as radiation dominated.

**1 Introduction**

In the standard electroweak theory, the baryon violating processes are suppressed for temperatures below the electroweak scale ( $\approx 100$  GeV). Therefore the currently observed baryon asymmetry was finally determined at this temperature scale. Since for these temperatures infrared singularities render perturbation theory uncertain <sup>1,2</sup>, a non-perturbative treatment of the plasma is needed, which can be provided by numerical lattice simulations. For this purpose we performed large scale lattice simulations of the SU(2) Higgs model in 4 Euclidean dimensions. Our results <sup>3-8</sup> also provide an estimation of the reliability <sup>9</sup> of the reduction step, which is used to relate simulation results obtained in effective models in three dimensions <sup>10,11</sup> to physics.

The results detailed in this talk are obtained with a Higgs boson mass of  $M_H \simeq 34$  GeV, for which a strong first order phase transition is observed at the electroweak scale. In the simulations we use the following action with inverse lattice spacings in the range  $2T_c \leq a^{-1} \leq 5T_c$

$$S[U, \varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2} \text{Tr} U_{pl}\right) + \sum_x \left\{ \frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) + \lambda \left[ \frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) - 1 \right]^2 - \kappa \sum_{\mu=1}^4 \text{Tr} (\varphi_{x+\hat{\mu}}^+ U_{x\mu} \varphi_x) \right\}. \quad (1)$$

Here  $U_{x,\mu}$  denotes the gauge link variable,  $U_{pl}$  the smallest Wilson loop and  $\varphi_x$  is the scalar field in  $2 \otimes 2$  isospin matrix notation.

**2 Lines of constant physics and critical temperature**

To keep the renormalised couplings  $g_R$  and  $\lambda_R$  constant when stepping down in the lattice spacing  $a$ , we guess the bare  $\beta$  and  $\lambda$  from the 1-loop renormalisation

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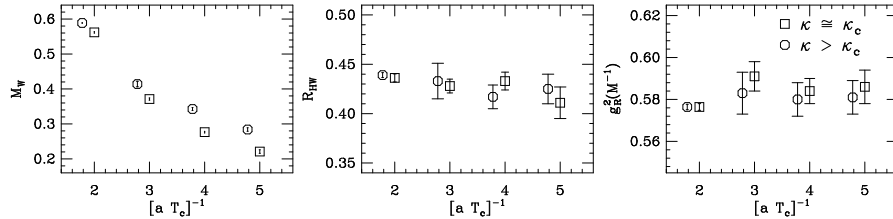


Figure 1: Flow of the renormalised parameters with the lattice spacing. For  $R_{HW} := M_H/M_W$  we have  $g_R R_{HW} = (32\lambda_R)^{1/2}$ . The octagons refer to points with  $\kappa$  shifted by  $5 \cdot 10^{-5}$  resp.  $4 \cdot 10^{-5}$  in case of the finest lattice, when compared to the squares.

group equation. The hopping parameter  $\kappa$  is tuned to its critical value in a simulation at a temporal lattice extent  $L_t$ , as detailed in Ref. <sup>5</sup>. Which determines the critical temperature to be  $aT_c = L_t^{-1}$ .

In order to obtain the size of the lattice spacing in physical units, the Higgs and W-boson masses and the renormalised coupling<sup>3</sup> are measured at  $T = 0$ . In a detailed finite volume study at  $a^{-1} = 2T_c$  we find a spatial lattice extent  $L_s \geq 12$ , which relates to the correlation length of the Higgs boson as  $L_s \geq 2.9\xi_H$ , to be sufficient to keep the renormalised parameters constant within an accuracy of 1% or even better<sup>5</sup>. Using these physical volumes we investigate the influence of the finite  $a$  and the uncertainty of the critical hopping parameter on the renormalised parameters. The outcome is shown in figure 1, where a small shift of  $\kappa$  affects only the W-mass in lattice units. This leads to a significant contribution to the error of  $a$  from the uncertainty of  $\kappa$ . The renormalised couplings  $\lambda_R$  and  $g_R$  however are unaffected within their small errors. A second important result is that the renormalised couplings are unchanged when the lattice spacing is decreased. This means that our simulations indeed follow the lines of constant physics.

We are now in the position to give a precise estimate of the critical temperature in physical units. This is shown in figure 2. The results from the previous paragraph are given by the white symbols. Their error bars contain the uncertainties of  $\kappa_c$  and of both of the  $M_H$  values. Since the lattice artifacts of eq. (1) are  $\mathcal{O}(a^2)$ , we extrapolate with a quadratic ansatz to small  $a$ -values, which is justified from the good  $\chi^2 \simeq 1$ . The result  $T_c/M_H|_{a \rightarrow 0} = 2.15(4)$  is shown by the filled symbol. Note the surprisingly small scaling violations of only 5%, when comparing the result at  $a^{-1} = 2T_c$  to the extrapolated value. With  $M_W \simeq 80$  GeV we get  $T_c = 72.8 \pm 1.3$  GeV for  $a \rightarrow 0$ .

The perturbative estimate<sup>2</sup> shown by the dashed line, differs from the extrapolated lattice result by three standard deviations, so one cannot exclude

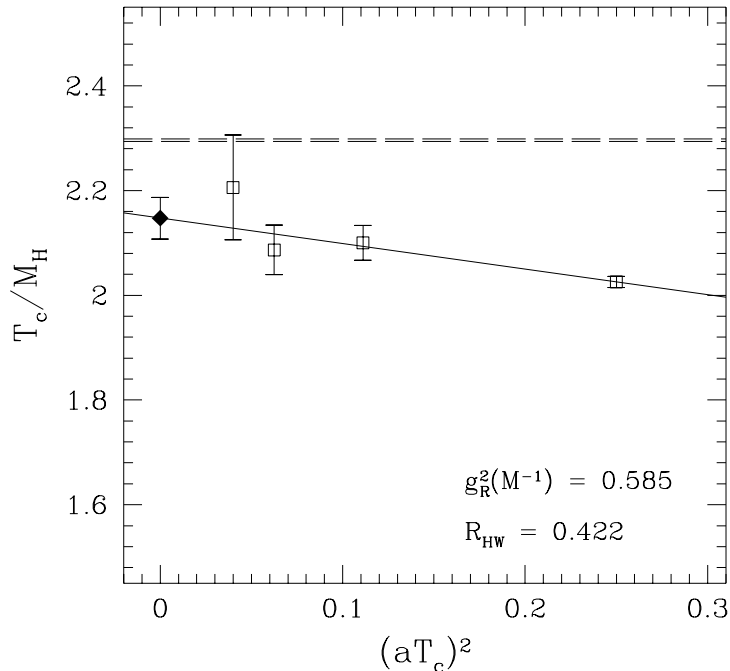


Figure 2: Extrapolation of  $T_c/M_H$  to the continuum limit which is given by the filled symbol. The dashed lines represents the perturbative predictions<sup>2</sup>.

non-negligible higher order or non-perturbative contributions.

### 3 Scalar field vacuum expectation value

The scalar field vacuum expectation value  $v$  plays a prominent rôle in the semi-classical estimation of the sphaleron rate in the symmetry broken phase. A gauge invariant renormalised vacuum expectation value is defined by  $v_R := 2M_W/g_R$ . However since the mass determination is demanding in terms of CPU time, we approximate this by  $v(T) := [2\kappa (\rho_x^2(T) - \rho_{x,\text{sym}}^2(T_c))]^{1/2}$  with  $\rho_x^2 := \frac{1}{2}\text{Tr}(\varphi_x^\dagger \varphi_x)$ . It has been checked that  $v_R$  and  $v(T)$  are in reasonable agreement for  $T = 0$  and  $T \approx T_c$ <sup>7</sup>. The result for  $v/T$  is given in figure 3. We also given an estimate of the supercooling obtained in the thin wall approximation. From this one can expect at most an increase of 15% in the exponent of the sphaleron rate due to supercooling. With the Clausius-Clapeyron Equa-

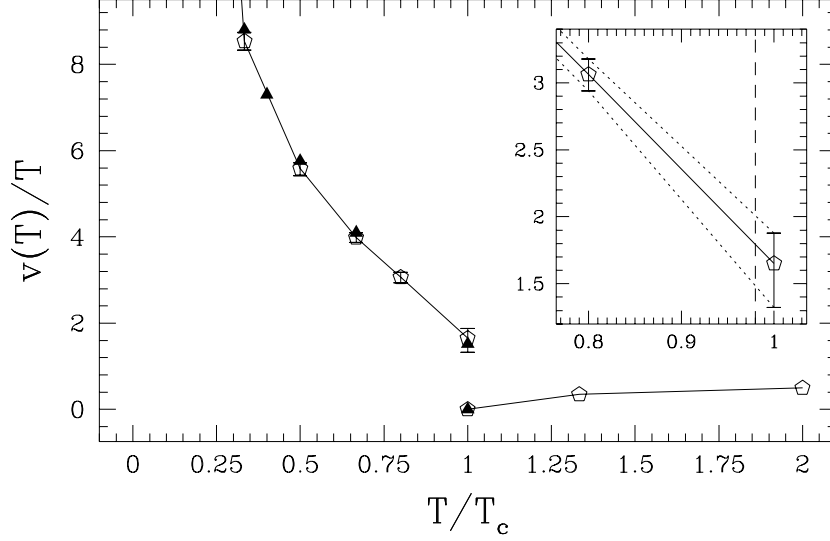


Figure 3: Result for  $v/T$  as a function of temperature. Filled triangles denote results for  $a^{-1} = 2T_c$ , the pentagons those for  $4T_c$ . The error bars in the enlargement give the uncertainty of  $\kappa_c$  only. The vertical dashed line gives the thin wall estimate of the supercooling<sup>7</sup>.

tion<sup>2</sup> the latent heat can be estimated from the discontinuity at  $T_c$ .

$$\Delta\epsilon/T_c^4 = 0.281(19)|_{a^{-1}=2T_c}, \quad \Delta\epsilon/T_c^4 = 0.31(12)|_{a^{-1}=4T_c}. \quad (2)$$

#### 4 Thermodynamics of the plasma

In this section we discuss the thermodynamic quantity  $\delta := \frac{1}{3}\epsilon - P$  which measures the deviation of the plasma from pure radiation. With  $\epsilon$  the energy density and with  $P$  the pressure is denoted, while  $\delta$  can be determined from

$$\frac{\delta}{T^4} = \frac{(L_t)^4}{3} \left[ 8 \frac{\partial \kappa}{\partial \tau} \langle L_\varphi \rangle - \frac{\partial \lambda}{\partial \tau} \langle (\rho_x^2 - 1)^2 \rangle - 6 \frac{\partial \beta}{\partial \tau} \langle 1 - \frac{1}{2} \text{Tr} U_{pl} \rangle \right] \Big|_{g_R, \lambda_R}, \quad (3)$$

with  $\tau := -\log(M_W)$  and  $L_\varphi := \frac{1}{2} \text{Tr}(\varphi_{x+\hat{\mu}}^+ U_{x,\mu} \varphi_x)$ . We determine the derivatives of  $\beta$  and  $\lambda$  using the 1-loop renormalisation group equations and those of  $\kappa$  from fits to the simulation results for  $\kappa_c$ <sup>5</sup>. Since eq. (3) contains divergent vacuum contributions,  $\delta(T=0)$  must be subtracted to obtain the physical result, which is show in figure 4. The figure shows different errors: statistical errors by vertical bars, uncertainty of  $\kappa_c$  by horizontal bars and the uncertainty

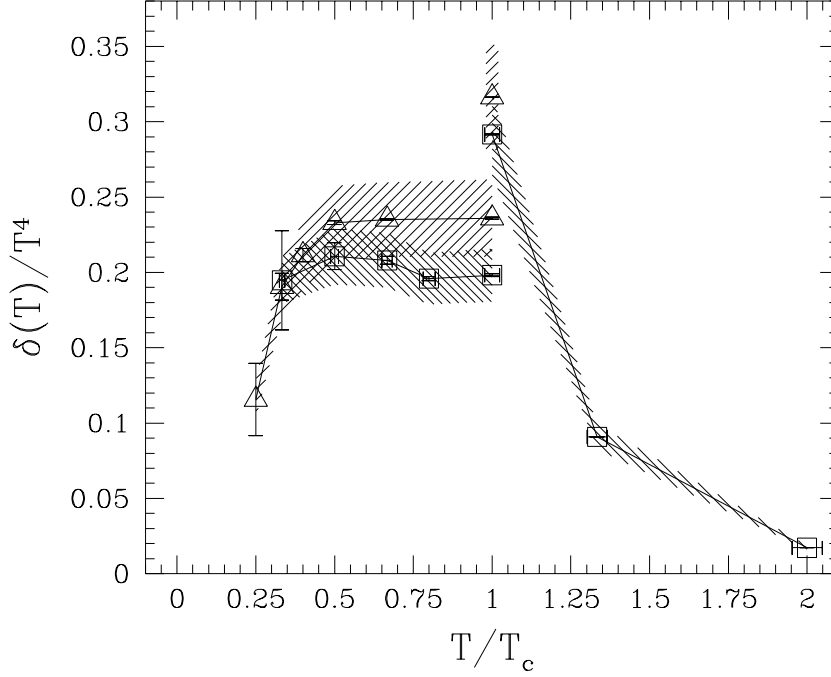


Figure 4: Result for  $(\frac{1}{3}\epsilon - P)T^{-4}$ . Triangles represent  $a^{-1} = 2T_c$ , squares  $4T_c$ .

arising from  $\frac{\partial \kappa}{\partial \tau}$  by the shaded region. In the symmetric phase  $\delta/T$  goes down by one order of magnitude when  $T$  is increased from  $T_c$  to  $2T_c$  where it is almost compatible with 0. This strong decay is confirmed by the good agreement of the slopes  $\frac{\partial \delta/T}{\partial \kappa}$  for both  $a$ -values<sup>5</sup>, so that for  $T \geq 2T_c$  the plasma can be considered to be radiation dominated.

Again from the jump at  $T_c$  the latent heat can be determined:

$$\Delta\epsilon/T_c^4 = 0.240(30 + 4)|_{a^{-1}=2T_c}, \quad \Delta\epsilon/T_c^4 = 0.28(3 + 9)|_{a^{-1}=4T_c}. \quad (4)$$

The first number in the brackets represents the statistical error and the second one the uncertainty of  $\kappa_c$ . The result is in agreement to eq. (2) and no scaling violation is to be observed between the results at  $a^{-1} = 2T_c$  and  $4T_c$ .

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